Subject: Physics
Course: US03CPHY01
Optics
UNIT- I Geometrical Optics

Introduction to lens systems:

A lens is made of glass and is bounded by two regular curved surfaces. Lens is an image forming device. It forms an image by refraction of light at its two bounding surfaces. A single lens with two refracting surfaces is a simple lens.

As it is easy to make spherical surfaces, most of the lenses are made of spherical surfaces and have a wide range of curvatures. Single lens are rarely used for image formation because they suffer from various defects. In optical instruments, such as microscopes, telescopes, camera etc. the combination of lenses are used for forming images of objects. An optical system consists of number of lenses placed apart, and having a common principal axis. The image formed by such a coaxial optical system is good and almost free of aberrations.

Cardinal points of lens systems:

In 1841 Gauss showed that any number of coaxial lenses can be treated as a single unit. The lens makers' formula can be applied to measure the distances from two hypothetical parallel planes. The points of intersection of these planes with the axis are called the principal points or Gauss points.

There are six points: (i) Two focal points, (ii) Two principal points and (iii) Two nodal points. These six points are known as Cardinal points of an optical system. The planes passing through these points and which are perpendicular to the principal axis are known as cardinal planes. We can find the image of any object without making a detailed study of the passage of the rays through the system using cardinal points.

(1) Principal points and Principal planes:

- Consider an optical system having its principal focal points F_1 and F_2. A ray OA travelng parallel to the principal axis and incident at A is brought to focus at F_2 in the image space as shown in the given figure (3).

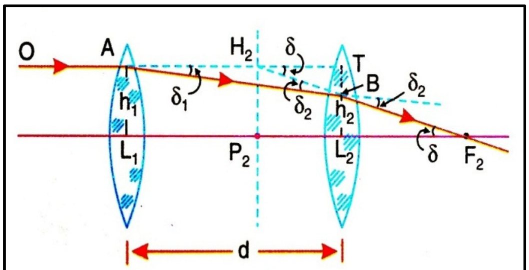

Figure : (3)

The actual ray is refracted at each surface of the optical system and follows the path $OABF_2$. If we extend the incident ray OA forward and the emergent ray BF_2 backward, they meet each other at H_2. A plane drawn through the point H_2 and perpendicular to the principal axis. This plane is called the **principal plane of the optical system**.

H_2P_2 is the principal plane in the image space and is called the **second principal plane**. The point P_2 at which the second principal plane intersects the axis, is called the **second principal point**.

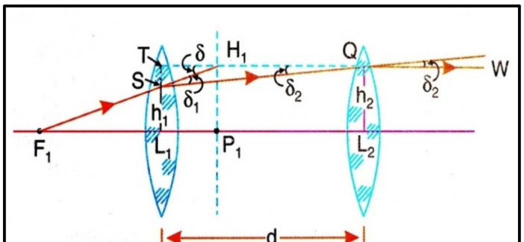

Figure : (4)

- Now consider figure (4), we can locate the principal plane H_1P_1 and principal point P_1 in the object space. Consider the ray F_1S passing through the first principal focus F_1 such that after refraction it emerges along QW parallel to the axis (as in figure (3). The rays F_1S and QW intersect at H_1. A plane perpendicular to the axis and passing through H_1 is called the **first principal plane**. The point of intersection P_1 of the first principal plane with the axis is called the **first principal point.**

- **Conjugate planes:** Consider figure (5) the two incident rays (F_1S & OA) are directed towards H_1 and after refraction seem to come from H_2. Therefore, H_2 is the image of H_1. Thus H_1 and H_2 are the conjugate points and the planes H_1P_1 and H_2P_2 are pair of **conjugate planes.** It has also been observed that $\mathbf{H_2P_2 = H_1P_1}$.

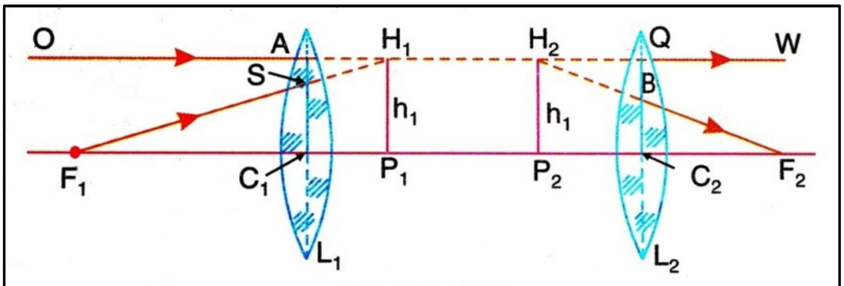

Figure : (5)

Thus lateral magnification of the planes is +1. Thus, the first and second principal planes are planes of unit magnification and are therefore called as **unit planes** and points P_1 and P_2 are called **unit points**. Note that principal planes are conceptual planes and do not have physical existence within the optical system.

(2) **Focal Points and Focal Planes:**

When a parallel beam of light traveling from the left to the right (in object space) is incident on the optical system then the beam comes together at a point F_2, on the other side (in image space) of the optical system. The beam passes through the point F_2 is called the **second focal point** of the optical system**.**

A beam of light passing the point F_1 on the axis of the object side is travel parallel to the axis after emergence through the optical system as shown in figure (5). This point F_1 is called the **first focal point.**

Thus the focal points can be defined as

(i) **The first focal point:** The first focal point is a point on the principal axis of optical system such that a beam of light passing through it is travel parallel to the principal axis after refraction through the optical system.

(ii) **The second focal point:** The second focal point is a point on the principal axis of the optical system such that a beam of light traveling parallel to the principal axis of the optical system passes through it after refraction through the system.

The planes passing through the principal focal points F_1 &F_2 and perpendicular to the axis are called first focal plane and second focal plane respectively.

Properties of focal planes:

(i) The main property of the focal planes is that the rays starting from a point in the focal plane in the object space corresponds to a set of conjugate parallel rays in the image space. Similarly, a set of parallel rays in the object space corresponds to a set of rays intersecting at a point in the focal plane in the image space.

(ii) The distance of the first focal point from the first principal point, i.e. F_1P_1 is called the *first focal length f_1* of the optical system and the distance of the second focal point from the second principal point F_2P_2 is called the *second focal length f_2*. f_1 and f_2 are also known as the focal lengths in the object space and image space respectively.

(iii) When the medium is same on the two sides of the optical system $f_1=f_2$

(3) Nodal Points and Nodal Planes:

Nodal points are the points on the principal axis of the optical system where light rays, without refraction, intersect the optic axis. The planes passing through the nodal points and perpendicular to the principal axis are called the **nodal planes**. Where as the principal planes are the planes where all refractions are assumed to occur and the nodal planes are planes where refraction does not take place. Figure (6) represents an optical system with the help of its cardinal points.

A ray of light AN_1 directed towards one of the nodal points N_1, after

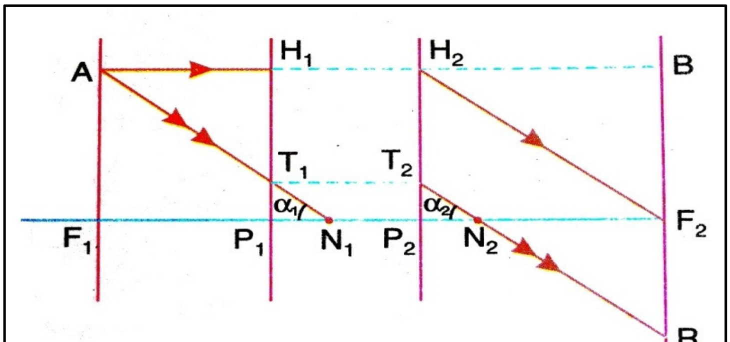

Figure : (6)

refraction through the optical system along N_1N_2, the ray emerges out from the second nodal point N_2 in a direction N_2R parallel to the incident ray. **The distances of the nodal points are measured from the focal points.**

Properties of nodal points and nodal planes:

(i) **The nodal points are a pair of conjugate points on the axis having unit positive angular magnification**

Let H_1P_1 and H_2P_2 be the first and second principal planes of an optical system. Let AF_1 and BF_2 be the first and second focal plane respectively. Consider a point a situated on the first focal plane. From A draw a ray AH_1 parallel to the axis. The conjugate ray will proceed from H_2, appoint in the second principal plane such that $H_2P_2 = H_1P_1$ and will pass through the second focus.

Take another ray AT_1 parallel to the emergent ray H_2F_2 and incident on the first principal plane at T_1. It will emerge out from T_2, a point on the second principal plane such that $T_2P_2 = T_1P_1$, and will proceed parallel to the ray H_2F_2. The point of intersection of the incident ray AT_1 and the conjugate emergent ray T_2R with the axis give the positions of the nodal points. It is clear that two points N_1 and N_2 are a pair of conjugate points and the incident ray AN_1 is parallel to the conjugate emergent ray T_2R.

Further, $$\tan\alpha_1 = \tan\alpha_2$$

The ratio $\frac{\tan\alpha_2}{\tan\alpha_1} = \gamma$ represents the angular magnification.

$$\therefore \frac{\tan\alpha_2}{\tan\alpha_1} = 1 \qquad (12)$$

Therefore, the points N_1 and N_2 are a pair of conjugate points on the axis having unit positive angular magnification.

(ii) The distance between two nodal points is always equal to the distance between two principal points

In figure (6), we see that in the right angled Δ^{les} $T_1P_1N_1$ and $T_2P_2N_2$

$$T_1P_1 = T_2P_2$$
$$\angle T_1P_1N_1 = \angle T_2P_2N_2 = \alpha$$

Therefore, the two Δ^{les} are congruent.

$$\therefore\ P_1N_1 = P_2N_2$$

Adding N_1P_2 to both the sides, we get

$$\therefore\ P_1N_1 + N_1P_2 = P_2N_2 + N_1P_2$$
$$\therefore\ P_1P_2 = N_1N_2 \qquad (13)$$

Thus, the distance between the principal points N_1 and N_2 is equal to the distance between the principal points P_1 and P_2.

(iii) The nodal points N_1 and N_2 coincide with the principal points P_1 and P_2 respectively whenever the refractive indices on either side of the lens are the same.

Now consider the two right angled Δ^{les} $T_1P_1N_1$ and $T_2P_2N_2$ in figure (6).

$$AF_1 = H_2P_2$$
$$\angle AN_1F_1 = \angle H_2F_2P_2$$

$\therefore$ The two Δ^{les} are congruent.

$$F_1N_1 = P_2F_2$$

But
$$F_1N_1 = F_1P_1 + P_1N_1$$
$$\therefore F_1P_1 + P_1N_1 = P_2F_2$$
$$\therefore P_1N_1 = P_2F_2 - F_1P_1$$

Also

$$P_2F_2 = +f_2 \text{ and } P_1F_1 = -f_1$$
$$\therefore P_1N_1 = P_2N_2 = (f_1 + f_2)$$

As the medium is the same, say air, on both the side of the system

$$f_2 = -f_1$$
$$\therefore P_1N_1 = P_2N_2 = 0 \qquad (14)$$

Thus, the principal points coincide with the nodal points when the optical system is situated in the same medium.

Construction of image using cardinal points:

The cardinal points are used to construct the image corresponding to any object placed on the principal axis of the system.

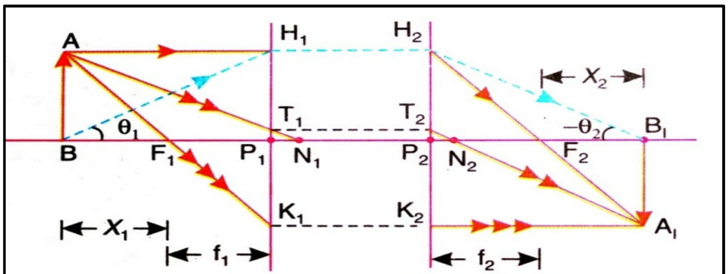

Figure : (7)

Suppose F_1, F_2 be the principal foci, P_1,P_2 the principal points and N_1,N_2 the nodal points of the optical system as shown in the given figure (7). AB is a linear object on the axis. Now in order to find the image of the point A we make the following construction.

i) A ray AH_1, is drawn parallel to the axis touching the first principal plane at H_1. The conjugate ray will proceed from H_2 a point on the second principal plane such that $H_2P_2 = H_1P_1$ and will pass through the second principal focus F_2.

ii) A second ray AF_1K_1 is drawn passing through the first principal focus F_1 and touching the first principal plane at K_1. Its conjugate ray will proceed from K_2 such that $K_2P_2 = K_1P_1$ and it will be parallel to the axis.

iii) A third ray $A_1T_1N_1$ is drawn which is directed towards the first nodal point N_1. This ray passes after refraction through N_2 in a direction parallel to AN_1.

The point of interaction of any of the above two refracted rays will give the image of A. Let it be A_1. The perpendicular is drawn from A_1 on to the axis gives the image A_1B_1 of the object AB.

Newton's formula:

Consider the given figure (7), it is seen that $\Delta^{les}\ ABF_1$ and $F_1K_1P_1$ are similar

$$\frac{K_1P_1}{AB} = \frac{P_1F_1}{BF_1}$$

But $K_1P_1 = A_1B_1$

$$\therefore\ \frac{A_1B_1}{AB} = \frac{f_1}{x_1} \tag{15}$$

Further , $\Delta^{les}\ A_1B_1F_1$ and $H_2P_2F_2$ are similar

$$\frac{A_1B_1}{H_2P_2} = \frac{B_1F_2}{P_2F_2}$$

But $H_2P_2 = AB$

$$\frac{A_1B_1}{AB} = \frac{x_2}{f_2} \tag{16}$$

From equation (15) and (16) we get

$$\therefore\ \frac{h_2}{h_1} = \frac{f_1}{x_1} = \frac{x_2}{f_2} \tag{17}$$

or
$$x_1x_2 = f_1f_2 \tag{18}$$

This is the **Newton's formula**.

Combination of two thin lenses

The cardinal points of a coaxial optical system is determined by assuming first that the object at infinity.

- **When object at infinity:** Here we consider an object located at infinity, see given figure.

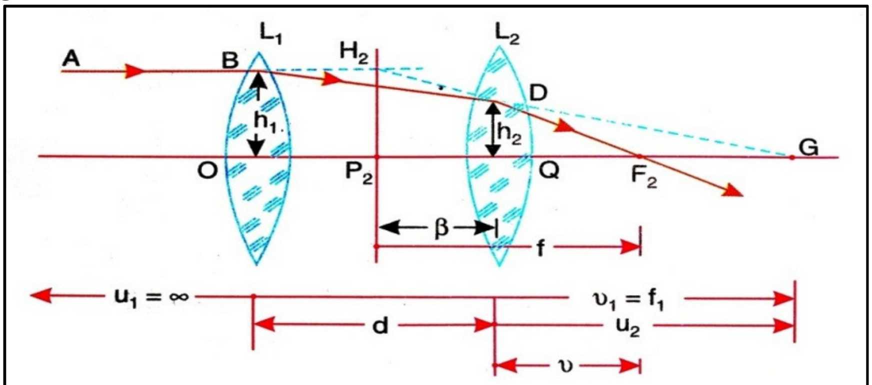

Figure : (8)

AB is a ray of light coming from an object situated at a very large distance, such that $u_1 = \infty$. The lens L_1 forms an image at G. However, because of the presence of the second lens L_2. G becomes the virtual object for L_2. The ray BD, instead of going along BDG, refracts along the path DF_2. When the ray AB is produced forward and the ray DF_2 backward, they intersect at H_2. The plane H_2P_2 normal to the axis may considered as the plane at which the refraction occurred and the plane is called **principal plane.**

- **Focal length of the system:**

Now we can write the expression for the refraction taking place at the surface of first lens as follows:

$$\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1} \qquad \therefore \frac{1}{OG}-\frac{1}{u_1}=\frac{1}{f_1}$$

As $u_1 = \infty$, we obtain OG =f_1

The equation for the refraction at the second lens may be written as

$$\therefore \frac{1}{v}-\frac{1}{u_2}=\frac{1}{f_2} \qquad \therefore \frac{1}{QF_2}-\frac{1}{QG}=\frac{1}{f_2}$$

or
$$\therefore \frac{1}{QF_2}=\frac{1}{f_2}+\frac{1}{f_1-d} \qquad \therefore \frac{1}{QF_2}=\frac{f_1+f_2-d}{f_2\left(f_1-d\right)} \tag{19}$$

The Δ^{les} BOG and DQG are similar and also the Δ^{les} $H_2P_2F_2$ and DQF_2 are similar

$$\therefore \frac{BO}{OG}=\frac{DQ}{QG} \qquad \therefore \frac{h_1}{f_1}=\frac{h_2}{(f_1-d)} \tag{20}$$

$$\therefore \frac{H_2P_2}{P_2F_2}=\frac{DQ}{QF_2} \qquad \therefore \frac{h_1}{f}=\frac{h_2}{QF_2} \tag{21}$$

From equation (19), (20) and (21) we get

$$\frac{h_1}{h_2}=\frac{f_1}{(f_1-d)}=\frac{f(f_1+f_2-d)}{f_2(f_1-d)}$$

or

$$\frac{1}{f}=\frac{f_1+f_2-d}{f_1f_2} \tag{22}$$

$$\therefore \frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2} \tag{23}$$

The location of focal point F_2 is determined by QF_2, which is known from the equation, the position of the principal plane P_2 is specified by the value of f calculated from equation (23).

- **Cardinal points**

(i) Second principal point :

Let us say the second principal plane H_2P_2 is located at a distance of $L_2P_2 = \beta$ from the second lens L_2. According to sign convention β would be negative as it is measured toward the left of lens.

$$\therefore QF_2 = f-(-\beta) = f+\beta$$

We can determine β using the equation for *f* into the above relation. Thus,

$$f+\beta=\frac{f_2(f_1-d)}{f_1+f_2-d}$$

$$\beta=-f+\frac{f_2(f_1-d)}{f_1+f_2-d}=\frac{-f_1f_2}{\Delta}+\frac{f_2f_1-f_2d}{\Delta}$$

Where $\Delta= f_1+f_2-d$

$$\therefore\ \beta=-f_2\frac{d}{\Delta} \tag{24}$$

or

$$\beta=-\frac{f_2d}{f_1+f_2-d} \tag{25}$$

But from eq. (22), we have $f_1+f_2-d=\frac{f_1f_2}{f}$

$$\beta=-\frac{fd}{f_1} \tag{26}$$

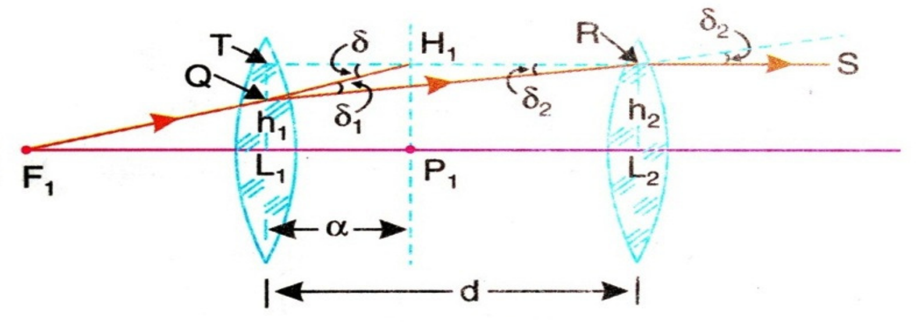

Figure : (9)

(ii) First principal point :

By considering a ray of light parallel to the axis and incident on the second lens L_2 from the right side, we can show that the distance of first principal plane. $L_1P_1=\alpha$ from the first lens L_1 is given by

$$\alpha=f_1\frac{d}{\Delta} \tag{27}$$

Also,

$$\alpha=+\frac{fd}{f_2} \tag{28}$$

This is the same as the result.

(iii) Second focal point :

Referring to figure (8), the distance of the second focal point F_2 from the second lens L_2 is given by

$$L_2F_2=P_2F_2-P_2L_2$$

$$=f-(-L_2P_2)=f+\beta$$

$$=f+\left(-\frac{fd}{f_1}\right)$$

$$\therefore\ L_2F_2=f\left(1-\frac{d}{f_1}\right) \tag{29}$$

(iv) First focal point :

The distance of the first focal point F_1 from the first lens $L_1$is given by

$$L_1F_1 = P_1F_1 - P_1L_1$$

$$= -f - (-L_1P_1) = -f + \alpha$$

$$= -f + \left(\frac{fd}{f_2}\right)$$

$$\therefore\ L_1F_1 = -f\left(1 - \frac{d}{f_2}\right) \tag{30}$$

(v) and (vi) nodal points :

As the optical system is considered to be located in air, P_1 and P_2 are also the positions of nodal points N_1 and N_2 respectively.

Lens Aberrations

The imperfect quality of the image formed by lens is known as lens ***aberration*** or the deviation of real images from the ideal images, in respect of the actual size, shape and position are called ***aberrations***. In actual practice the objects are bigger and a lens is required to produce a bright and magnified image.

The wide angle rays from the centre of the object and also the upper and lower parts of the object and falling near the top and bottom of the lens. These rays are known as peripheral or marginal rays.

In general, (a) peripheral rays of light do not meet at a single point after refraction through the lens. (b) the refractive index and hence the focal length of a lens is different for different wave lengths of the light.

For a given lens, the refractive index for violet light is more than that for red light.

There are two types of aberrations:

(1) **Monochromatic aberration:** The defects due to wide-angle incidence and peripheral incidence or aberration for monochromatic light, are called monochromatic aberrations.
(2) **Chromatic aberration:** The aberrations occur due to dispersion of light are called chromatic aberrations.

Types of monochromatic aberration:

Monochromatic aberrations are again divided in to five types and they are

(a) Spherical aberrations
(b) Coma
(c) Astigmatism
(d) Curvature of field
(e) Distortion

Spherical aberration

Definition: The peripheral rays passing through a lens are refracted more and come to focus closer to the lens. Paraxial rays passing through the lens are refracted less and come to focus away from the lens. Therefore, rays passing through different zones of a lens surface come to focus at different points. An image formed by paraxial rays will be surrounded by the image of peripheral rays and the image is blurred. **This phenomenon is known as Spherical aberration.**

Theory: The spherical aberration in the image formed by a single lens is shown in figure (10). Here O is a point object on the axis of the lens and I_p and I_m are the images formed by the paraxial and marginal rays respectively. In figure it is very clearly that paraxial rays of light from the image at a longer distance from the lens than the marginal rays.

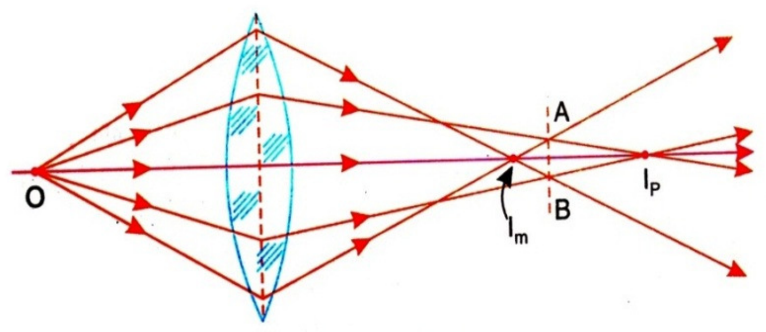

Figure : (10)

The image is not sharp at any point on the axis. If the screen is placed perpendicular to the axis at AB, the image appears to be a circular patch of diameter AB. At positions on the two sides of AB, the image patch has a large diameter. The patch of diameter AB is called the **circle of least con fusion**, which corresponds to the position of the best image.

The distance I_m I_p measures the **longitudinal spherical aberration**. The radius of the circle of least confusion measures the **lateral spherical aberration**.

Thus, for an object point O on the axis, the image extends over the length I_m I_p. This effect is called **spherical aberration**. Figure (11) shows the spherical aberration produced by a concave lens.

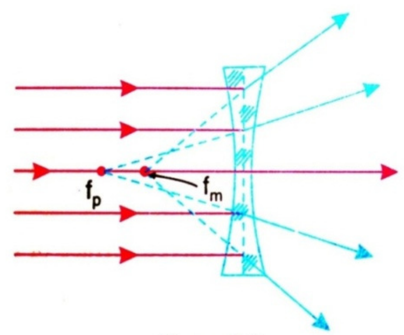

Figure : (11)

The spherical aberration produced by a lens depends on the distance of the object point and varies approximately as the square of the distance of the object ray above the axis of the lens. The **spherical aberration** produced by a **convex lens** is **positive** and that produced by a **concave lens** is **negative.**

- **Methods for reducing spherical aberration:**

(i) Spherical aberration can be minimized by using stops, which reduces the effective lens aperture. The stop used can be such as to permit either the axial rays of light or the marginal rays of light. However, as the amount of light passing through the lens is reduced, correspondingly the image appears less bright.

(ii) The conditions for minimum spherical aberration is

$$\frac{dx}{dk} = 0 \qquad (31)$$

where x is the longitudinal spherical aberration and,

$$k = \frac{R_1}{R_2}$$

Where, R1 and R2 are the radii of curvature. Now, differentiation of equation of spherical aberration gives

$$k=\frac{R_1}{R_2}=\frac{\mu(2\mu-1)-4}{\mu(2\mu+1)} \tag{32}$$

For a lens of refractive index μ=1.5 we get k= - 1/6. Thus, the lens, which produces minimum spherical aberration, is biconcave and the radius of curvature of the surface facing the incident light is one-sixth the radius of curvature of the other face. In general, the more curved surface of the lens should face the incident beam of light. Lens whose $\frac{R_1}{R_2}=-\frac{1}{6}$ is called a **cross lens**. A crossed lens is shown in figure (12).

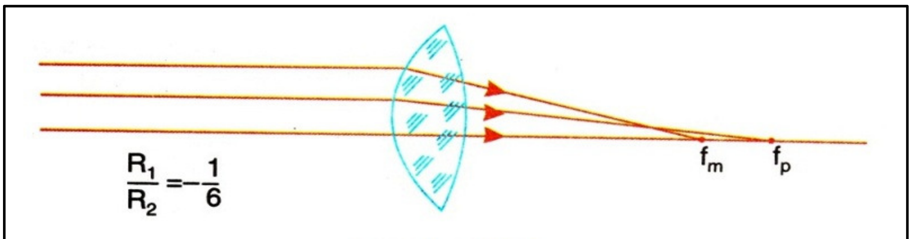

Figure : (12)

(iii) Plano-convex lenses are used in optical instruments so as to reduce the spherical aberration. When the curved surface of the lens faces the incident light, the spherical aberration is minimum. The spherical aberration in crossed lens $\frac{R_1}{R_2}=-\frac{1}{6}$ is only 8% less than that of a plano-convex lens having the same focal length and radius of the lens aperture. This is the reason why plano-convex lenses are generally used in place of crossed lenses without increasing the spherical aberration appreciably. Figure (13) represents the variation of longitudinal spherical aberration with the radius of the lens aperture for lenses of the same focal length and refractive index.

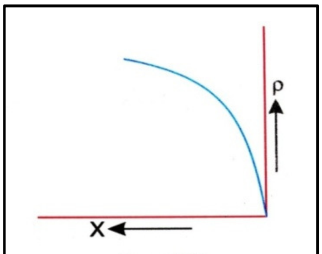

Figure : (13)

The spherical aberration will be very large if the plane surface faces the incident light. The spherical aberration is a result of larger deviation of the marginal rays than the paraxial rays. If the deviation of the marginal rays of light is made minimum, the focus f_m for a parallel incident beam will shift towards f_p , the focus for the paraxial rays of light and the spherical aberration will be minimum.

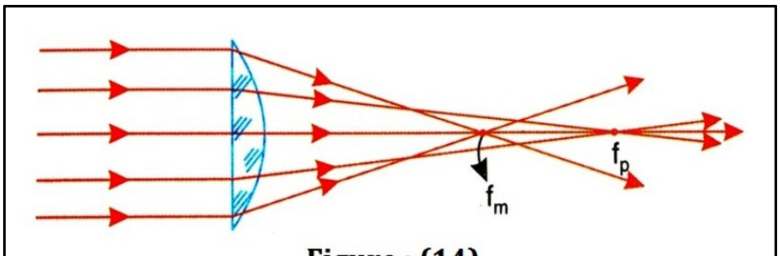

Figure : (14)

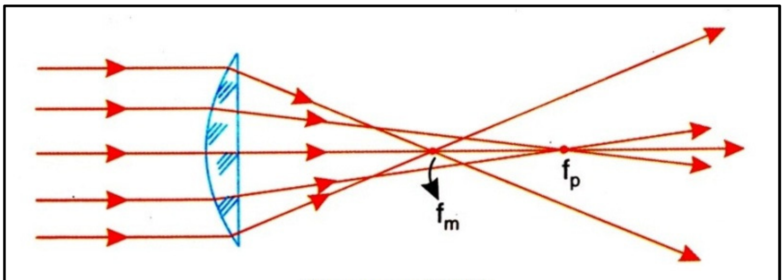

Figure : (15)

In plano-convex lens, when the plane surface faces the parallel beam of light , the deviation is produced only at the curved surface and hence the longitudinal spherical aberration (figure (14)) is more than when the curved surface faces the incident light, as shown in figure (15).

The spherical aberration produced by a by a single lens can be minimized by choosing proper radii of curvature.The shape factor q of the lens is given by $q = \frac{R_1 + R_2}{R_2 - R_1}$

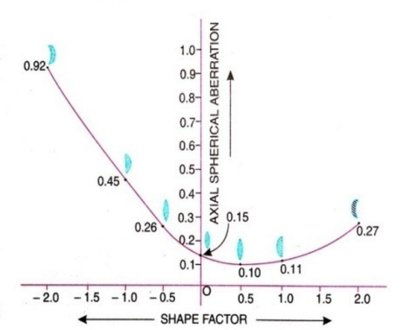

Figure : (16)

The spherical aberration for a double convex lens (shape factor q=0.5) is a minimum when the surface of smaller radius of curvature faces the incident parallel light. The spherical aberration for a plano-convex lens (shape factor q=+1.0) when the curved surface faces the incident light is only slightly more than the double convex lens. Hence, plano-convex lenses are preferred.

(iv) Spherical aberration can also be made minimum by using two plano-convex lenses separated by a distance equal to the difference in their focal length. In this arrangement, the two lenses equally share the total deviation and the spherical aberration is minimum. In figure (17), two plano-convex lenses of focal length f_1 and f_2 are separated by a distance d.

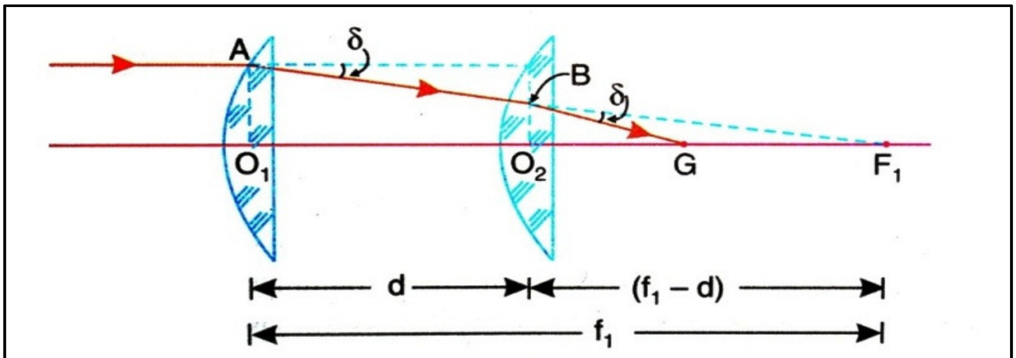

Figure : (17)

Let δ be the angle of deviation produced by each lens see figure (17).

$\angle BF_1G = \delta$, $\angle F_1BG = \delta$

And from triangle BGF_1, $BG = GF_1$ or $O_2G = GF_1$ (approximately)

$$O_2G = \frac{1}{2}(O_2F_1) = \frac{1}{2}(f_1 - d)$$

For the second lens F_1 is the virtual object and G is the real image. Substituting these values of object and image distances in formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_2}$$

$$\frac{1}{O_2G}-\frac{1}{O_2f_1}=\frac{1}{f_2}$$

$$\frac{2}{(f_1-d)}-\frac{1}{(f_1-d)}=\frac{1}{f_2}$$

$$\frac{1}{(f_1-d)}=\frac{1}{f_2}$$

$$f_2=f_1-d \quad or \quad d=f_1-f_2 \tag{33}$$

(v) Thus, the condition for minimum spherical aberration is that the distance between the two lenses is equal to the difference in their focal lengths

(vi) Spherical aberration for a convex lens is positive and that for a concave lens is negative. By a suitable combination of convex and concave lenses spherical aberration can be made minimum.

(vii) Spherical aberration may be minimized by using axial –GRIN lenses.

Coma

Definition: When object point is not situated on the axis then aberration produced by the lens is called Coma.

Theory: In the case of Spherical aberration, the image is a circle of varying diameter along the axis and in the case of Comatic aberration the image is comet shaped and hence the name is coma.

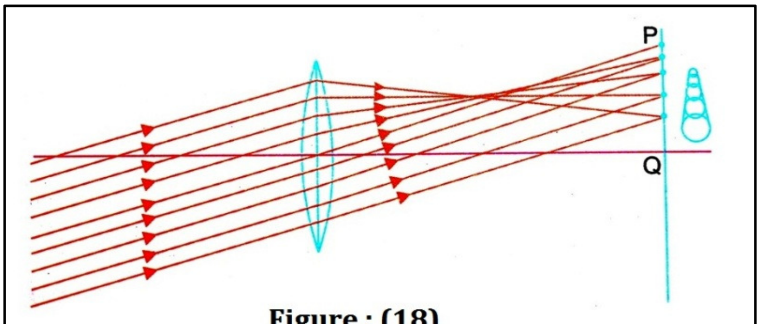

Figure : (18)

The effect of coma when the resultant image of a point off the axis is shown on the right side of the figure (18). The rays of light in the tangential plane are represented in the figure.

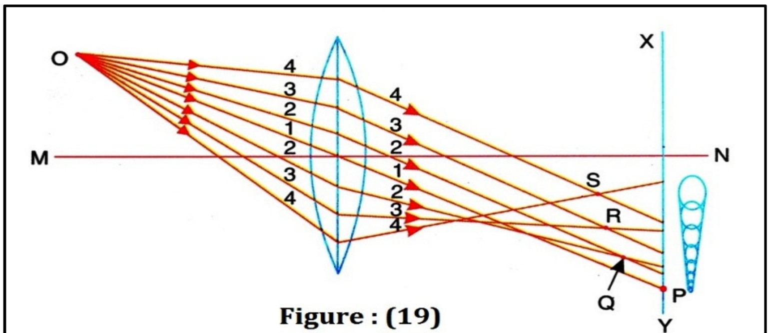

Figure : (19)

The presence of coma in the image due to a point object O situated off the axis of the lens is shown in above figure (19).The rays of light getting refracted through the centre of the lens(ray 1) meet the screen XY at the point P. rays 2.2, 3.3, etc getting refracted through the outer zones of the lens come to focus at points Q,R,S, etc. , nearer the lens and overlapping circular patches of gradually increasing diameter are formed on the screen. The resultant image of the point is comet-shaped as shown on the right side of the figure.

Let 1, 2, 3 etc., be the various zones of the lens See the figure (20a). Rays of light getting refracted through these different zones give rise to circular patches of light 1',2',3', etc. The screen is placed perpendicular to the axis of the lens and at the position where the central rays come to focus as in Figure (20b).

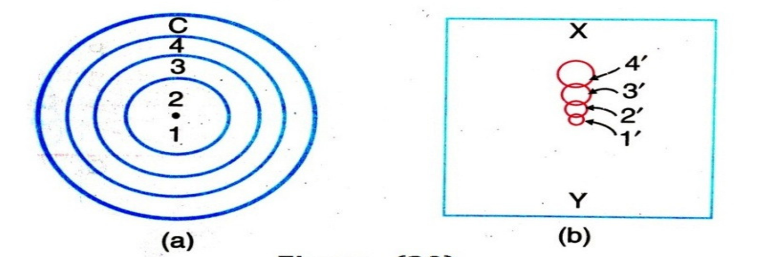

Figure : (20)

Coma is the result of varying magnification for rays refracted through different zones of the lens. In figure (19), rays of light getting refracted through the outer zones come to focus at points nearer the lens. Hence the magnification of the image due to the outer zones is larger than the inner zones and in this case coma is said to be **positive**. If the magnification produced in an image due to the other zones is similar, coma is said to be **negative**.

Methods of reduction:

(1) The coma can be corrected by properly choosing the radii of curvature of the lens surface. A lens corrected for coma will not be free from spherical aberration and the one corrected for spherical aberration will not be free from coma.

(2) Use of a stop or a diaphragm at the proper position eliminates coma.

(3) According to abbe, a German optician, coma can be eliminated if a lens satisfies the Abbe's sine condition viz.

$$\mu_1 y_1 \sin\theta_1 = \mu_2 y_2 \sin\theta_2 \tag{34}$$

Where μ_1, y_1 & θ_1 refer to the refractive index, height of the object above the axis, and the stop angle of the incident ray of light respectively. Similarly μ_2, y_2 & θ_2 refer to the corresponding quantities in the image space. The magnification of the image is given by $\frac{y_2}{y_1}$

$$\frac{y_2}{y_1} = \frac{\mu_1 \sin\theta_1}{\mu_2 \sin\theta_2} \tag{35}$$

Elimination of coma is possible if the lateral magnification $\frac{y_2}{y_1}$ is the same for all rays of light. Thus coma can be eliminated if, $\frac{\sin\theta_1}{\sin\theta_2}$ is a constant because $\frac{\mu_1}{\mu_2}$ is constant.

(4) A lens that satisfies the above condition is called an ***aplanatic*** lens.

Astigmatism

Definition: Astigmatism is the aberration in the image formed by a lens when the object points is off the axis. The difference between Astigmatism and coma is that in coma the spreading of the image takes place in a plane perpendicular to the lens axis and in Astigmatism the spreading takes place along the lens axis.

Theory:

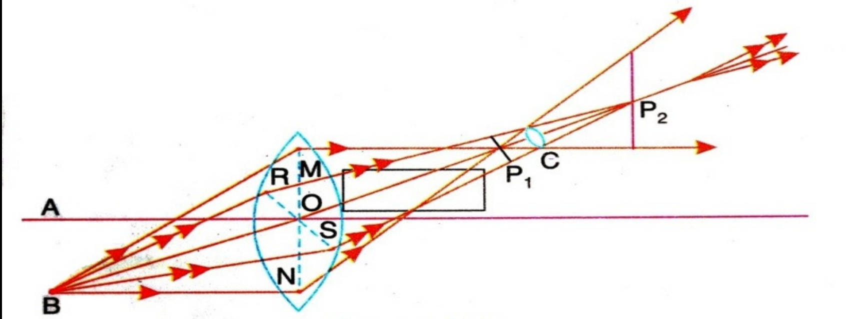

Figure : (21)

Figure (21), illustrates the defect of Astigmatism in the image of a point B situated off the axis. The cone of the rays of light refracted through the tangential (vertical) plane BMN comes to focus at point P_1 nearer the lens and the cone of rays refracted through the sagitial (horizontal) plane BRS comes to focus at the point P_2 away from the lens. All rays pass through a horizontal line passing through P_1 called **primary image** and through a vertical line passing through P_2 called **secondary image**. The refracted beam has an elliptical cross-section, which ends to a horizontal line at P_1 and a vertical line at P_2.the cross section of the refracted beam is circular at some point between the primary and secondary images and this is called the **circle of least confusion**. If a screen is held perpendicular to the refracted beam between the points P_1 and P_2 the shape of the image at different positions is as shown in the figure (22) here.

Figure : (22)

The primary and the secondary image surfaces and the surface of the best focus are illustrated in the figure (23).

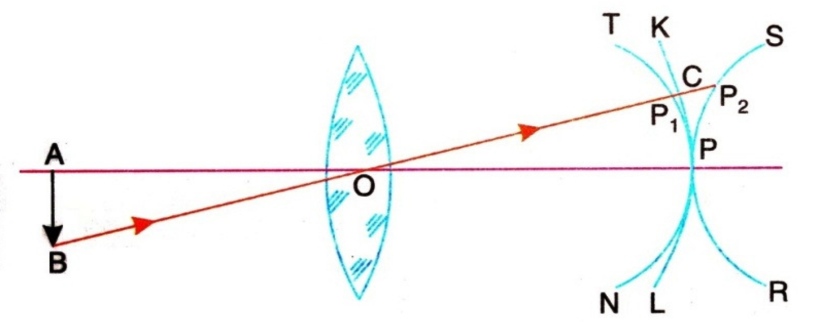

Figure : (23)

P_1 and P_2 are the images of object point B . TPN and SPR are the first and the second image surfaces and KPL is the surface of the best focus. The three surfaces touch at the point P on the axis. Generally the surface of the best focus is not plane but curved as shown. This defect is called the curvature of the field. The shape of the image surfaces depends on the shape of the lens and the position of the stops.

If the primary image surface is to the left of the secondary image surface then it is called **positive astigmatism**, and if the primary image surface is to the right of the secondary image surface then it is called **negative astigmatism**.

Method of reduction:

By using a convex and a concave lens of suitable focal length and separated by a distance, it is possible to minimize the astigmatic difference and such a lens combination is called an ***anastigmat***.

Curvature of the field

Definition: The image of an extended object due to a single lens is not a flat one but it will be a curved surface. The central portion of the image nearer the axis is in focus but the outer regions of the image away from the axis are blurred. The paraxial focal length is greater than the marginal focal length. **This defect is called the curvature of the field.**

This aberration is present even if the aperture of the lens is reduced by a suitable stop. The figure illustrates the presence of curvature of the field in the image formed by a convex lens. A real image formed by a convex lens curves towards the lens (fig.24) and a virtual image curves away from the lens (fig.25)

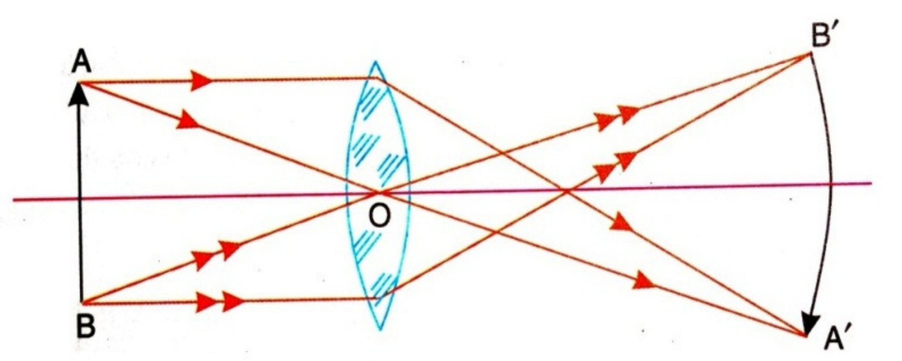

Figure : (24)

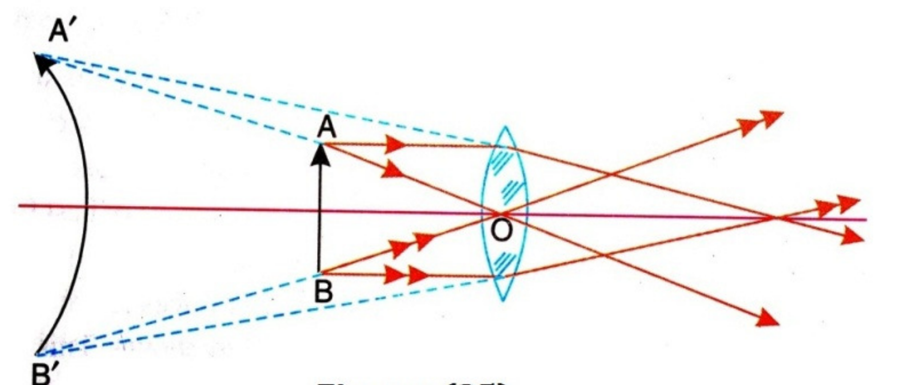

Figure : (25)

Figure (26) represents the curvature of the field present in the image formed by a curvature of the field present in the image formed by a concave lens.

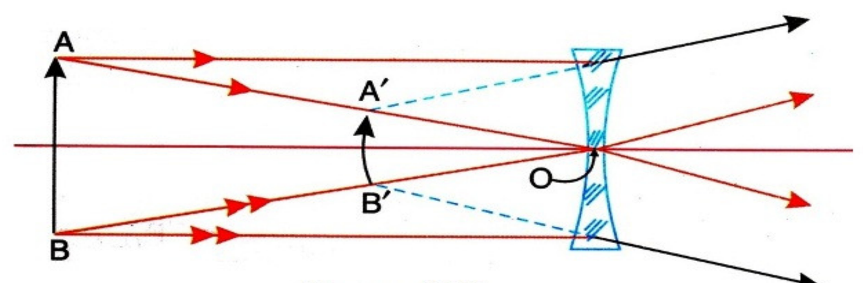

Figure : (26)

For a system of thin lenses, the curvature of the final image can, theoretically, be given by the expression

$$\frac{1}{R} = \sum \frac{1}{\mu_n f_n} \tag{36}$$

Where R is the radius of the final image, μ_n and f_n are the refractive index and focal length of the n^{th} lens. For the image to be flat, R must be infinity

$$\therefore \quad \frac{1}{R} = \sum \frac{1}{\mu_n f_n} = \frac{1}{\infty} = 0$$

Correspondingly, the condition for two lenses placed in air, reduces to

$$\frac{1}{\mu_1 f_1} + \frac{1}{\mu_2 f_2} = 0 \tag{37}$$

This is known as Petzwal's condition for no curvature. This condition holds good whether the lenses are separated by a distance or placed in contact. As the refractive indices μ_1 and μ_2 are positive, the above condition will be satisfied if the lenses are of opposite sign. If one of the lenses is convex the other must be concave.

Distortion

Definition: The variation in the magnification produced by a lens for different axial distances produce an aberration is called **distortion.**

There are two types of distortion:

(1) pi-cushion distortion and (2) barrel-shaped distortion.

In pi-cushion distortion, the magnification increases with increasing axial distance and the image of an object is shown in figure (29 b).

In barrel-shaped distortion, the magnification decreases with increasing the axial distance and the image of an object is shown in figure (23 c).

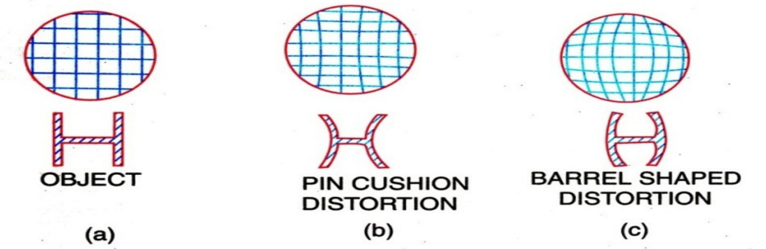

Figure : (29)

In the case optical instruments, a little amount of distortion may be present but it must be completely eliminated in a photographic camera lens, where the magnification of the various regions of the object must be the same.

In the absence of stops, which limit the cone of rays or light striking the lens, a single lens is free from distortion. But, if stops are used, the resulting image is distorted.

If a stop is placed before the lens the distortion is barrel-shaped see figure (30) and if stop is placed after the lens, the distortion is pin-cushion type see figure (31).

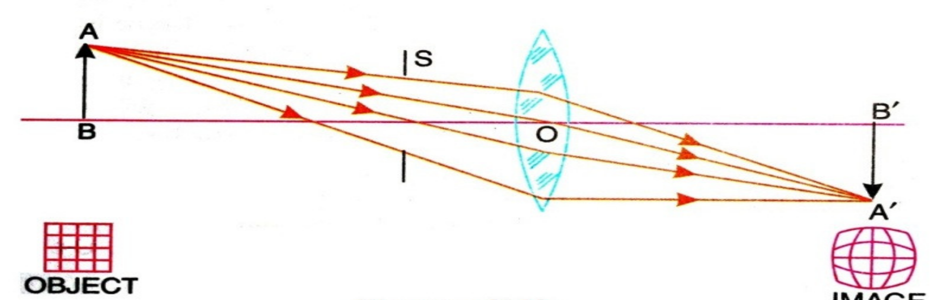

Figure : (30)

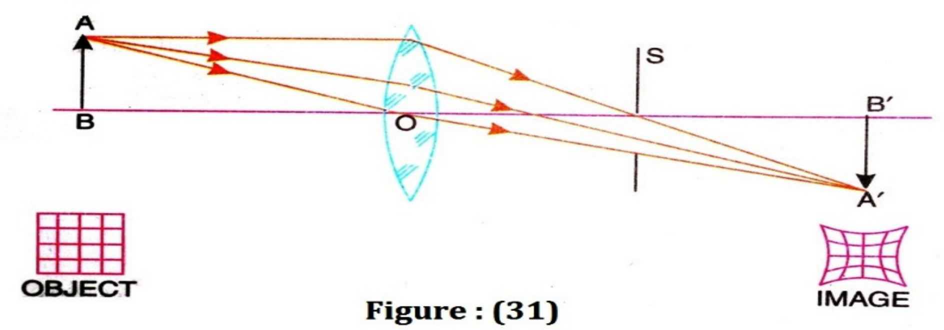

Figure : (31)

To eliminate distortion a stop is placed between two symmetrical lenses, so that the pin-cushion distortion produced by the first lens is compensated by the barrel-shaped distortion produced by the second lens see figure (32). Projection and camera lenses are constructed in this way.

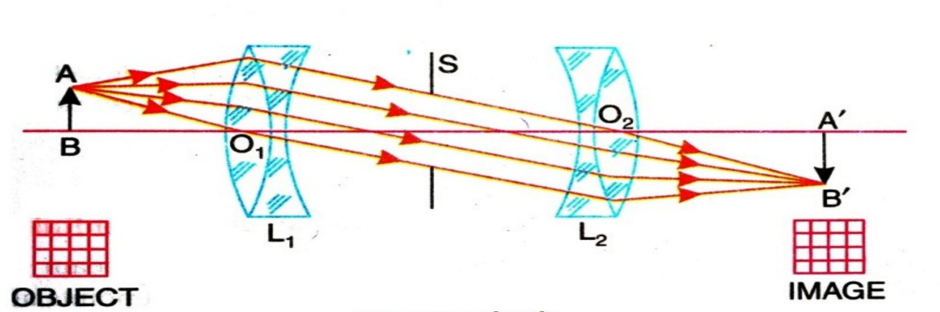

Figure : (32)

Chromatic aberration in a lens

Definition: A single lens produces a coloured image of object illuminated by white light and this defect is called chromatic aberration. Elimination of this defect in a system of lenses is called ***achromatism***.

(a) **Expression for longitudinal chromatic aberration for an object at infinity.**

When a parallel beam of white light is passed through a lens, the beam gets dispersed and rays of light of different colours come to focus at different points along the axis. The violet rays of light come to focus at a point nearer the lens and the red rays of light at a farther point see figure (33). f_v is the focus for the violet rays and f_r is the focus for the red rays. The colours in between violet and red come to focus between f_v and f_r. The distance $(f_r - f_v) = x$ is called the **longitudinal or axial chromatic aberration**.

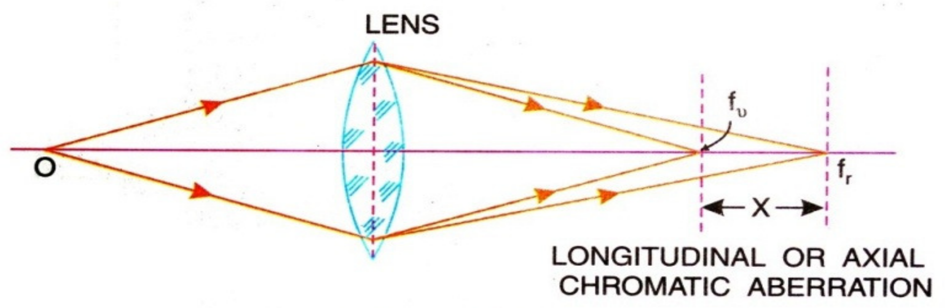

Figure : (33)

The focal length of a lens is given by

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{38}$$

or

$$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{f(\mu - 1)} \tag{39}$$

Similarly

$$\frac{1}{f_v} = (\mu_v - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{40}$$

$$\frac{1}{f_v} = \frac{(\mu_v - 1)}{(\mu - 1)f} \tag{41}$$

$$\frac{1}{f_r} = (\mu_r - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{42}$$

$$\frac{1}{f_r} = \frac{(\mu_r - 1)}{(\mu - 1)f} \tag{43}$$

From equation (41) and (43), we get

$$\frac{1}{f_v} - \frac{1}{f_r} = \frac{1}{(\mu - 1)f}(\mu_v - 1 - \mu_r + 1)$$

$$\frac{f_r - f_v}{f_v f_r} = \frac{(\mu_v - \mu_r)}{(\mu - 1)f}$$

Taking, $f_v f_r = f^2$, where f is the mean focal length and no one can write

$$\frac{f_r - f_v}{f^2} = \frac{(\mu_v - \mu_r)}{(\mu - 1)f}$$

$$f_r - f_v = \frac{(\mu_v - \mu_r)f}{(\mu - 1)}$$

$$f_r - f_v = \omega f \tag{44}$$

Where $\omega = \frac{(\mu_v - \mu_r)}{(\mu - 1)}$ is known as the dispersive power of the material.

Thus the axial chromatic aberration for a thin lens for an object at an infinity is equal to the product of the dispersive power of the material of the lens and f is the mean focal length. It is also clear that a single lens cannot form an image free from chromatic aberration.

(b) Expression for the longitudinal chromatic aberration for an object at finite distance:

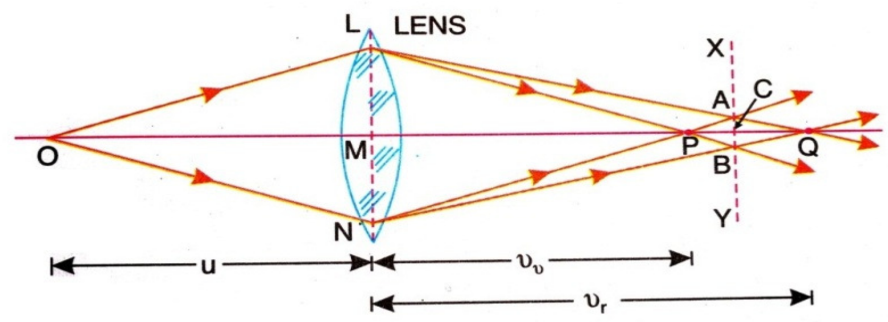

Figure : (34)

Consider a point object illuminated by white light and situated on the axis of the lens. Coloured images are formed along the axis. The violet image is nearest the lens and the red image is the farthest. In between these two images as shown in given figure (34), if a screen is placed at the position XY, the image of least chromatic aberration is formed.

Let u be the distance of the object and v_v and v_r the distance of the violet and red images on the axis of the lens. If f_v and f_r are the focal lengths for the violet and red rays if the light then

$$\frac{1}{v_v} - \frac{1}{u} = \frac{1}{f_v} \tag{45}$$

$$\frac{1}{v_r} - \frac{1}{u} = \frac{1}{f_r} \tag{46}$$

Subtracting eq. (46) from (45)

$$\frac{1}{v_v} - \frac{1}{v_r} = \frac{1}{f_v} - \frac{1}{f_r}$$

$$\frac{v_r - v_v}{v_v v_r} = \frac{f_r - f_v}{f_r f_v}$$

Taking, $v_v v_r = v^2$, and $f_v\, f_r = f^2$

$$\frac{v_r - v_v}{v^2} = \frac{f_r - f_v}{f^2}$$

But, $f_r - f_v = \omega f$,

$$\frac{v_r - v_v}{v^2} = \frac{\omega f}{f^2} = \frac{\omega}{f}$$

So
$$v_r - v_v = \frac{\omega v^2}{f} \tag{47}$$

It is clearly seen in this case that the longitudinal chromatic aberration depends on the distance of the image and hence on the distance of the object from the lens, in addition to its dependence on the dispersive power and the focal length of the lens.

Eyepieces

• Importance of an objective lens

The magnification power of a simple microscope can be increased by decreasing the focal length of the lens. However, the focal length of a lens cannot be decreased beyond a certain limit. The lens of a small focal length has a smaller diameter because the curvature of the surface is large and the field of view is small.

Therefore, to increase the magnifying power, two separate lenses are used. The lens near the object is called **objective**, which forms a real image of an object as shown in figure (35).

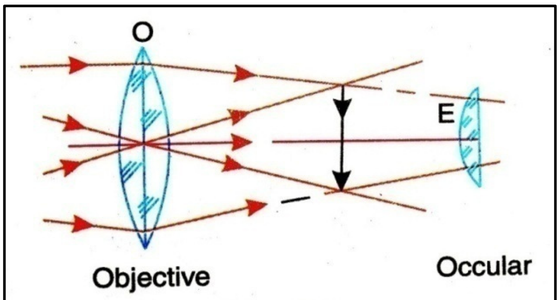

Figure : (35)

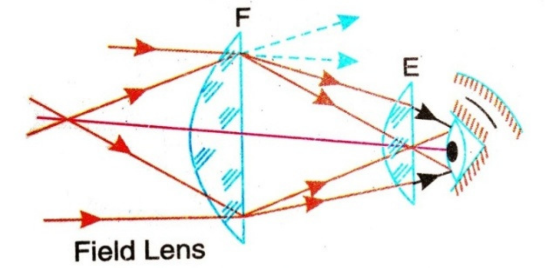

Figure : (36)

The lens used to enlarge this image viewed by the eye, is called **eyepiece or ocular.**

An optical instrument is required to produce a magnified image free from aberration and a bright image covering a wide field of view. If a single lens is used as an eyepiece, the final image will suffer from spherical and chromatic aberrations.

Another drawback is the small field of view, which becomes lesser and lesser as the magnification of the instrument is increased, The rays passing through the outer portions of the image are refracted through the peripheral portions of the eye lens and they cannot enter the small aperture of the pupil of the eye placed closed to the eye lens see figure (35).

Hence only that part of the image which is nearer to the axis, will be seen. Therefore, the final image will cover a small field of view.

To increase the field of view and magnifying power an extra lens introduced between objective and eye lens is called a **field lens.** The function of the field lens is to gather more of the rays from the objective toward the axis of the eye piece see figure (36).

The field lens and the eye lens together constitute an ocular or eyepiece. Two lens are made and kept in such a way that their combination is achromatic and free from spherical aberrations.

There are two types of eyepieces (i) Huygens and (ii) Ramsden eyepiece.

Huygens eyepiece

In the Huygens eyepiece a converging beam enters the field lens and forms a virtual image before the eye lens.

- **Construction**

Huygens eyepiece consists of two lenses having focal lengths in the ratio 3:1 and the distance between them is equal to the difference in their focal lengths. The focal lengths and the positions of the two lenses as shown in figure (37). This eyepiece is free from chromatic as well as spherical aberration as it satisfies the two conditions simultaneously.

i) The lens combination acts as an achromatic system if $D = \frac{f_1 + f_2}{2}$, where D is the distance between the lenses and f_1 and f_2 are the focal length of the two lenses.

ii) The lenses produces equal deviations of the incident ray when the distance between the two lenses is equal to f_1-f_2

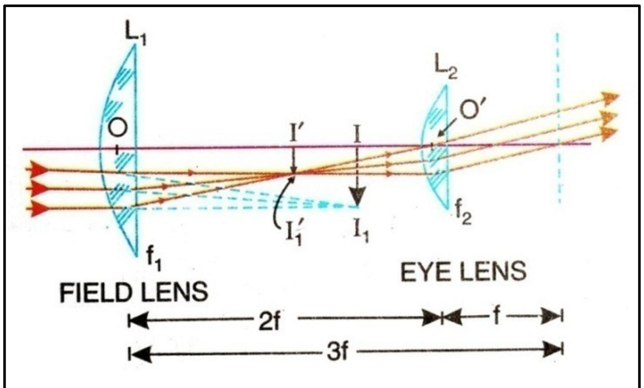

Figure : (37)

The field and the eye lenses used are plano convex and are placed with their convex surfaces towards the incident ray. In this way, the total deviation due to the combination is divided into four parts which makes the combination to have minimum spherical aberration.

In spherical and chromatic aberrations are to be minimized simultaneously, the following condition is to be satisfied.

$$D = \frac{f_1 + f_2}{2} \qquad \text{for Chromatic aberration}$$

$$D = f_1 - f_2 \qquad \text{for Spherical aberration}$$

Combining the two aberrations, we obtain

$$\frac{f_1 + f_2}{2} = f_1 - f_2$$

$$f_2 \ = 3 f_1$$

or

$$\therefore D = 3f_1 - f_1 = 2f_1 \tag{48}$$

Thus satisfy the conditions for minimum chromatic and spherical aberrations, the focal length of the field lens should be three times the focal length of the eye lens and the distance between them should be equal to twice the focal length of the eye lens. Huygens' eyepiece is constructed on this principle.

- **Theory**

The objective forms an image, which serves as a virtual object for the field lens. The field lens forms real inverse image I'I'$_1$. If this image is situated at the principal focus of the eye lens, then the final image is at infinity.

- **Equivalent focal length:**

The equivalent focal length of the eyepiece can be found as follows. If F is the equivalent focal length of the eyepiece, then it is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{3f} - \frac{2f}{3f^2} = \frac{2}{3f}$$

$$F = \frac{3f}{2} \tag{49}$$

The equivalent lens leis behind field lens at a distance of

$$x = \frac{d \times F}{f} = \frac{2f \times \frac{3f}{2}}{f} = 3f \quad (50)$$

In other words, the equivalent lens is at a distance of $3f - 2f = f$ behind the eye lens.

• Position of cross wire:

It has been observed that the principal focus of the equivalent lens lies f/2 before the eye lens. The image due to objective must be formed in order that the final image is at infinity. The rays coming from the objective are focus at I_1. It is here that the cross-wire should be placed. The image I I_1 is at a distance of f/2 from the eye lens or 3 f/2 from the field lens. Hence, for field lens u = -3 f/2. Then

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\therefore -\frac{2}{3f} + \frac{1}{v} = \frac{1}{3f}$$

or $$v = f \quad (51)$$

In other words, II_1 lies midway between the two lenses and so fixes the position of the cross wire or scale at this position. Since the image found by the objective lies behind the field lens, hence this eyepiece is called a **negative eyepiece**. It may be noted that the image of the scale is formed by the eye lens where as the final image is produced by both the lenses. Hence the image and the scale would not be magnified equally and so the measurements will not be reliable. The image of the cross wire or scale would have all the defects of an image formed by a single lens. Hence in instruments using Huygens' eyepieces, scale is not used.

• Merits and Demerits

i) Huygens' eyepiece is fully free from chromatic aberration because the distance between the lenses is equal to half the sum of their focal lengths.
ii) Spherical aberration is also minimum because the distance between the two lenses is equal to the difference of their focal lengths.
iii) The field of view of this eyepiece is smaller than that of Ramsden's eyepiece.

Cardinal points of Huygens eyepiece:

(i) First principal points:

We have equivalent focal length $F = \frac{3f}{2}$, f_2=*f and* f_1=*3f*

The equivalent lens should be placed at a distance α from the field lens, which is given by $$\propto = \frac{d \times F}{f_2} = \frac{2f \times \frac{3f}{2}}{f} = 3f$$

Since the distance between the field lens L_1 and the eye lens L_2 is *2f*, the position of equivalent lens is given by $3f - 2f = f$

i.e. it should be placed away from the eye lens as shown by dotted line at P1 as shown in figure. The first principal point P_1 lies at a distance $\propto = 3f$ from the field lens.

(ii) Second principal points:

The second principal point P_2 lies at a distance β from the eye lens towards the field lens and is given by $\beta = -\frac{d \times F}{f_1} = \frac{-2f \times \frac{3f}{2}}{3f} = -f$

Hence, the second principal point P_2 lies before a distance f from the eye lens.

(iii) First focal points:

The first focal point F_1 from lens L_1 is given by

$$L_1F_1 = -F\left(1-\frac{d}{f_2}\right)$$
$$= -\frac{3}{2}f\left(1-\frac{2f}{f}\right)$$
$$\therefore L_1F_1 = \frac{3}{2}f$$

Hence, first focal point F_1 lies at a distance $\frac{3}{2}f$ from the field lens L_1.

(iv) Second focal points:

The second focal point F_2 from lens L_2 is given by

$$L_2F_2 = F\left(1-\frac{d}{f_1}\right)$$
$$= \frac{3}{2}f\left(1-\frac{2f}{3f}\right)$$
$$\therefore L_2F_2 = \frac{f}{2}$$

Hence, second focal point F_2 lies at a distance $\frac{f}{2}$ from the eye lens L_2.

The cardinal points are shown in above figure.

Ramsden eyepiece

Ramsden's eyepiece consists of two plano convex lenses each of focal length f separated by a distance equal to (2/3) f . The lenses are kept with their curved surface facing each other, as shown in the figure (38), thereby reducing spherical aberration. The field lens is a little larger than the intermediate image and is placed close to this image to as much light as possible to pass through it. The eye lens has a smaller diameter but carries out the actual magnification.

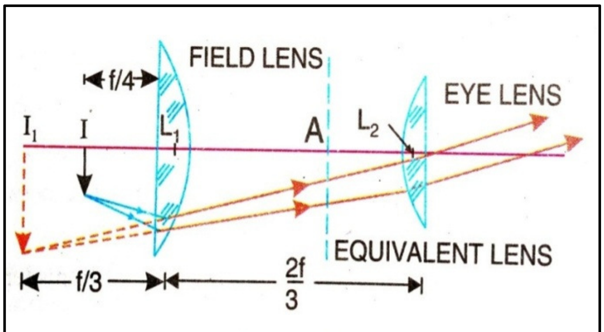

Figure : (38)

- **Theory**

The objective forms the real inverted image I of object. The field lens gives a virtual image I_1. I_1 is the object for the eye lens, which gives the final image at infinity, because I_1 made to lie at its principal focus.

- **Equivalent focal length:**

The equivalent focal length of the eyepiece can be found as follows. If F denotes the focal length of the equivalent lens, then

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}-\frac{\frac{2}{3}f}{f^2}=\frac{2}{3f}$$

$$=\frac{2}{f}-\frac{2}{3f}=\frac{4}{3f}$$

$$\therefore \quad F=\frac{3f}{4} \qquad (52)$$

The equivalent lens of focal length $F=\frac{3f}{4}$ must be placed behind the field lens at a

distance $$\alpha=\frac{F\times d}{f}=\frac{(\frac{3}{4})f\times(\frac{2}{3})f}{f}=\frac{f}{2} \qquad (53)$$

Thus the equivalent lens lies between the field lens and the eye lens.

- **Position of the cross wire:**

The cross wire should be placed at the position of I. Now, the position of I relative to field lens can be found as follows.

If the final image is to be formed at infinity, the image I should lie in the focal plane of the equivalent lens. In other words, the distance AI should be equal to F=(3 ƒ) / 4. Since AL_1 = ƒ/2, I_4 = ƒ/4. Therefore, the objective should produce the image at a distance of ƒ/4 in front of the field lens. A fine scale may be placed here. Since the scale and image would be magnified equally, the measurement would be true.

- **Merits and Demerits:**

i) The field of view of the eyepiece is fairly wide.
ii) It is not entirely free from chromatic aberration since the distance between the two lenses is not equal to half the sum of their focal lengths. However, chromatic aberration is minimized by using an achromatic combination both for the field lens and the eye lens.
iii) Spherical aberration is minimized by using two plano-convex lenses.

Ramsden's eyepiece is used practically in all instruments where measurements of the size of the final image are to be made. The eyepiece is sometimes referred to as **positive eyepiece** because a real image is formed by the objective in front of the field lens, which further acts as a real object for the eye lens.

Cardinal points of Ramsden eyepiece:

(i) First principal points:

We have equivalent focal length $F=\frac{3f}{4}$, $d=\frac{2f}{3}$ $f_2=f$ *and* $f_1=f$

The equivalent lens should be placed at a distance α from the field lens, which is given by $$\propto=\frac{d\times F}{f_2}=\frac{\frac{2}{3}f\times\frac{3f}{4}}{f}=\frac{f}{2}$$

Since the distance between the field lens L_1 and the eye lens L_2 is $d=\frac{2f}{3}$, the position of equivalent lens is given by $\frac{3f}{4}-\frac{f}{2}=\frac{f}{4}$

i.e. it should be placed before the eye lens at a distance $\frac{f}{4}$ as shown by dotted line at P1 as shown in figure. The first principal point P_1 lies at a distance $\propto=\frac{f}{2}$ from the field lens.

(ii) Second principal points:

The second principal point P_2 lies at a distance β from the eye lens towards the field lens and is given by $\beta = -\frac{d\times F}{f_1} = -\frac{\frac{2}{3}f\times\frac{3f}{4}}{f} = -\frac{f}{2}$

Hence, the second principal point P_2 lies before a distance $\frac{f}{2}$ from the eye lens.

(iii) First focal points:

The first focal point F_1 from lens L_1 is given by

$$L_1F_1 = -F\left(1-\frac{d}{f_2}\right)$$

$$= -\frac{3}{4}f\left(1-\frac{\frac{2}{3}f}{f}\right)$$

$$\therefore L_1F_1 = -\frac{f}{4}$$

Hence, first focal point F_1 lies before at a distance $-\frac{f}{4}$ from the field lens L_1.

(iv) Second focal points:

The second focal point F_2 from lens L_2 is given by

$$L_2F_2 = F\left(1-\frac{d}{f_1}\right)$$

$$= \frac{3}{4}f\left(1-\frac{\frac{2}{3}f}{f}\right)$$

$$\therefore L_2F_2 = \frac{f}{4}$$

Hence, second focal point F_2 lies at a distance $\frac{f}{4}$ from the eye lens L_2.

The cardinal points are shown in above figure.

Comparisons of Ramsden eyepiece and Huygens eyepiece

	Ramsden eyepiece	Huygens eyepiece
1	Ramsden eyepiece is positive eyepiece. The image formed by the objective lies in front of field lens. Therefore, cross-wire can be used.	Huygens eyepiece is negative eyepiece. The image formed by the objective lies in between the two lenses. Therefore, cross-wire cannot be used.
2	The condition for minimum spherical aberration is not satisfied. But by spreading the deviations over four surfaces , spherical aberration is minimized.	The condition for minimum spherical aberration is satisfied.
3	It does not satisfy the condition for achromatism but can be made achromatic by using an achromatic doublet as the eye lens.	It satisfies the condition for achromatism.
4	It is achromatic for only two chosen colours.	It is achromatic for all colour.
5	The other types of aberrations are better eliminated. Coma is absent and distortion is 5% higher.	The other types of aberrations like pincushion distortion are not eliminated.
6	The eye clearance is 5% higher,	The eye clearance is too small and less comfortable.